Sketches for the foundations of a contemporary experimental treatise on Harmony.

Prof. Dr. Marcus Alessi Bittencourt
Universidade Estadual de Maringá (UEM) - mabittencourt@uem.br

Resumo: Neste artigo é esboçada uma listagem de tópicos importantes a serem tratados na preparação de um tratado contemporâneo de Harmonia. O artigo dá um breve relato sobre a atual situação desta pesquisa em andamento, que tenta mapear conceitos harmônicos arquetípicos. São explicados alguns de seus fundamentos teóricos, tais como o uso do temperamento igual de doze sons por oitava como abstração do contínuo de alturas e o uso de conceitos da Teoria de Conjuntos, juntamente com conceitos mais experimentais como os de aspereza-classe, aspereza-Tn, tonicidade, fonicidade, azimute, comunalidade de alturas e vicinalidade, simetria e escoamento gravitacional. Tradução do título: Apontamentos para as bases de um tratado contemporâneo experimental de Harmonia.

Palavras-chave: Harmonia; Teoria de Conjuntos; Teoria de Costère; Psicoacústica.

Abstract: This paper sketches a listing of important topics to be addressed during the preparation of a contemporary treatise on Harmony. It reports briefly on the actual state of this ongoing research, which tries to map out archetypical harmonic concepts. Some of its theoretical fundamentals are explained, such as the use of the 12-tone equal temperament as an abstraction of the pitch-height continuum and the use of Set Theory concepts, alongside with the more experimental concepts of roughness-class, Tn-roughness, tonicity, phonicity, azimuth, pitch commonality and vicinity, symmetry, and gravitational flow.

Key-words: Harmony; Set Theory; Costère Theory; Psychoacoustics.

Introduction

Whenever a composer has to deal with the assemblage of musical ideas out of sounds of definite pitch, he must make decisions regarding the manipulation of vertical combinations of notes and the logic of their horizontal development. Here the composer must comprehend the expressive value and blending properties of simultaneities of notes and must formulate logical mechanisms for the successive progression of these sound amalgams. For centuries now – or even for more than a millennium – , these preoccupations have been congregated for study in the venerable discipline called Harmony, as we can see from definitions given by theorists such as Gioseffo Zarlino (1517-1590):

> "Proper [Harmony] is the one described by Lactantius Firmianus [(ca. 240 - ca. 320)] in his *De Opificio Dei* as follows: 'Musicians call harmony properly the concord of strings or voices that are consonant in their measures, without any offense of the ears,' meaning by this the concord that arises from the movements that the parts of each song make until they reach the end." (Zarlino 1571: II-12, 80; apud Blackburn 1987: 228).

One of the main problems when tackling the teaching of Composition nowadays is the decision of what to teach regarding the discipline of Harmony, as we just defined it. For this task, should we repeat the traditional views given by the numerous historical and modern academic Harmony treatises available? After all the musical experiments of the 20th century, the inconvenience of teaching old-fashioned out-of-context ideas is obvious. Do not get me wrong: in the great historical Harmony treatises we will, of course, find a magnificent wealth of extremely useful knowledge. The problem here is that when we face composition instruction this whole body of research sources can only, at their very best, establish how Harmony "used to be conceived". The zealous teacher-theorist can most certainly adapt and patch together to his needs and liking ideas from a myriad of sources, but this ends up turning Harmony instruction into some sort of a History of Music Theory class. This scenario may be fine and excellent for the training of musicologists but certainly lacks the objectivity and cohesion of a unified attempt of codification of Harmonic principles, one capable of inspiring new creative compositional work.

Another common approach is to address the discipline of Harmony by means of the analysis of works by important composers, mainly those from the 20th century. To base the instruction on the evaluation of any existing music, irrespective of its historical period, can only reveal solutions to specific Harmony problems, but will not help us in the task of clearly formulating those problems at their very origin. It is only after the formulation of primeval harmonic questions that concrete musical examples can be of any usefulness, as case studies of brilliant harmonic accomplishment.

All traditional approaches to Harmony, as well as the ones taken by individual composers or groups of them, are bodies of specific answers to basic problems arisen from the work with sounds of definite pitch. Since human beings are, for the most part, quite anatomically similar, we may assume that sounds physically do affect their senses in similar ways. In the same way that there is a common ground where people can relate regarding gustatory concepts such as sweet, sour, salty and bitter, there must be a common ground for sound perception as well. Of course, culture and individual adaptation will have a big say in the actual way in which a cook will blend his ingredients and in the amount of spices and flavors that eaters can indeed endure and appreciate. Nonetheless, there are basic truths about food ingredients that must be understood before any culinary practice can be developed. This is the very spirit behind the project I am sketching here of creating a contemporary Harmony treatise: to look for paradigmatic archetypical musical concepts regarding the vertical and horizontal blending of pitched sounds. These fundamental principles can not only help to explain how tradition came to be what it is (see, for example, Huron 2001) but can also help to enlighten the experimentation of contemporary composers, serving as grounds for the development of new ways of harmonic thinking.

In this paper I sketch a list of important topics to be addressed in the preparation of such a treatise on Harmony. The issues discussed here spring from the compilation of ideas contained in a great deal of historical and contemporary research, all the way from the Renaissance (such as Zarlino 1571) to the 19th century (Oettingen 1866 and Riemann 1893, for example), and to seminal 20th century musical theoretical works such as Hindemith 1945, Costère 1954, Levy 1985 and Forte 1973. It includes as well modern psychoacoustical research such as Parncutt 1989. The sketches here are by all means not finished, but they do nonetheless reveal the basic master plan of the research I am presently conducting.

The pitch continuum and the primeval intervals.

Nowadays, it is common for composers to experiment with microtonality, xenotonality and tuning systems, making full use of the pitch-height continuum. Nonetheless, the sensory consonance/dissonance relationships have been proven to be derived from the exact configuration of the frequency spectrum of the sounds used (see Sethares 1998). As long as Music continues to be made out of sounds with harmonic spectra, the just-intonation intervals contained in the first ten partials of the harmonic series will remain the most important measuring references for all work with Harmony. These intervals, which we can call primeval intervals for their archetypical importance, are, in order of decreasing perceptual intelligibility: the diapason (ratio 2:1, the perfect octave), the abstraction of the first, second, fourth, and eighth harmonics; the diapente (ratio 3:2, the 3-limit perfect fifth), the abstraction of the third and sixth harmonics; the sesquiquarta (ratio 5:4, the 5-limit major third), the abstraction of the fifth and tenth harmonics; the supertripartiens-quarta (ratio 7:4, the 7-limit minor seventh), the abstraction of the seventh harmonic; and the sesquioctava, or tonus (ratio 9:8, the 3-limit major second), the abstraction of the ninth harmonic. From the differences and combinations of these primeval intervals come the secondary intervals: the 5-limit just-intonation minor second (16:15), the 5-limit minor third (6:5), the 5-limit minor sixth (8:5), the 5-limit major sixth (5:3), the 5-limit major seventh (15:8), and the tritone.

The work with just-intonation intervals generates enharmonic incompatibilities and clashes which have historically been dealt with by means of temperament methods (see Barbour 1951). Agreeing with historical empirical opinions, modern experimental studies have found that small inharmonicities generated by individual variations in tuning of the partials of as much as 3% in frequency (about 50 cents) are indeed well tolerated by our perception (see Moore, Peters & Glasberg 1985). If we consider this and the usual and well-documented slight inharmonicity of the natural timbre of musical instruments, plus the everyday fact that even excellent musical performances involve some amount of mistuning and/or pitch vibrato, it is not surprising that temperament methods do make sense for practical use. It seems that our perception has learned to relate a certain range of intervals of very similar size to one basic just-intonation interval pattern. It is in this capacity that a twelve-tone equally-tempered interval can, for example, act as a representative of a just interval, although with some amount of a consciously perceived increase in sensory dissonance, or roughness.

With this is mind, it is probably not correct to speak of pitch-height in terms of one single dimension of perception. Instead, it would be more accurate to say that every non-tempered interval entails at least two dimensions of perception: a) it represents in our imagination the closest just-intonation primeval or secondary interval, inheriting the harmonic properties of its model in an inversely proportional way to its divergence from it; and b) it includes a "deformation" roughness sensation, which can be largely exploited musically for expressive purposes. Thus, it is possible to consider twelve-tone equal temperament as an interesting practical compromise for formulating archetypical harmonic properties of sound combinations. These properties would then be extended to their microtonal variations.

Set Theory and the twelve-tone equal temperament universe.

Once we accept the domain of 12-tone equal temperament as an interesting workable compromise, great part of the work will be to analyze all possible combinations of notes – which we can call sets or collections –, comparing and classifying them according to their characteristics and properties. Well, if we consider using only the notes existent on an 88-key piano, there will be 2^{88} - 1 combinations possible (the -1 is to discard the empty set). This totals the grand amount of 309,485,009,821,345,068,724,781,055 combinations. To make this work feasible, it will be obviously necessary to reduce the universe of sets possible discarding combinations that are considered "superfluous" because of their equivalence to another considered set. This consideration of equivalence will be possible with the use of concepts such as pitch-class (Forte 1973), voicing permutation, transposition, etc. Thus, if we discard from the set universe the sets which are either identical to another one after removal of repeated pitch-classes, or which constitute a different voicing permutation or transposition of another set, there remains 351 sets, a far much more palatable number, which constitute the different ways of subdividing the perfect octave interval (see Costère 1954). In Set Theory jargon, these constitute the 351 Tn-types (see Rahn 1980). We could further restrict this universe by adding the consideration of equivalence by inversion, which leaves us with 223 Tn/TnI-types (Rahn 1980). Indeed, Set Theory (see Forte 1973 and Rahn 1980) is an excellent tool for this task. Among the aspects of Set Theory to be considered for inclusion in the treatise should be its descriptive numerical model, which can serve as main analytical symbology, and its concepts of pitch-class, interval-class, normal form, interval vector, inclusion relationships (complementary, subset and superset relationships), as well as transposition, inversion, invariance, and isomeric (Z-relation) relationships, among others. After its description by Set Theory methods, each one of the 351 Tn-types can be analyzed regarding the more experimental properties of Tn-roughness, tonicity, phonicity, azimuth, pitch commonality and vicinity, symmetry, and gravitational flow.

Roughness-class and Tn-roughness.

Roughness is the measurement of the intensity of the rough sensation we experience when our hearing organ is under the effect of the very fast beatings produced by the interaction of adjacent partials of sonorities (see Helmholtz 1885). This phenomenon of roughness has been explained through research on the physiology of the inner ear and on the cognition mechanisms involved in the processing of the perceived stimuli. Modern scholarship has been evaluating the strong role and contribution of roughness in the construction of the cultural percepts of musical consonance and dissonance (Tenney 1960 and Parncutt 1989).

If sensory dissonance plays a very important role on the definition of musical consonance and dissonance, it is very desirable to somehow incorporate it into Music Theory as yet another meaningful means of comparison between pitch collections. Nonetheless, it is common for Music Theory to deal with generalized harmonic structures and concepts, independently of instrumentation, dynamics, and voicing, as seen from the very Tn-type concept. According to the psychoacoustic theories of roughness (see, for example, Plomp & Levelt 1965), we can immediately see some big conceptual incompatibility problems when positing a generalized roughness measurement for Tn-types: roughness is dependent on timbre, on specific pitch height, and relative loudness. Among musicians, there is widespread agreement that an interval does retain its musical identity and properties throughout all its different transpositions and octave placements. The very concepts of pitch-class and interval-class (Forte 1973) rely on the notions of transpositional equivalence and octave equivalence. For example, although we do notice that a perfect fifth very low in the bass range is sensorially quite rougher than a perfect fifth in the treble range, we never doubt that these two sonorities are but different implementations of the same interval-class. Also, the intuition of music theorists throughout History has considered that sonorities with identical interval content should sound equally consonant/dissonant (see Zarlino 1571). This springs from the idea that intervals retain their mentally attributed consonance value irrespective of their relative pitch level placements or orderings. This is the evidence that cultural experience and training has here shaped our understanding and handling of the musical sounds in a radical way, directing the interpretation of dissimilar sensory stimuli towards the generalization represented by the interval-class concept. Considering all this, I am proposing the conception of a roughness measurement for a Tn-type by means of the positing of two abstract models: the roughness-class and the Tn-roughness. These can serve only as basic parameters for sensory dissonance comparison between Tn-types and do not intend to represent the actual roughness measurement of a specific vertical implementation of such types.

The roughness-class is the average roughness for a directed interval, irrespective of its pitch-height level and of its variations in timbre and relative loudness. In my proposed model, the average roughness of a directed interval is the calculated sensory roughness using the Hutchinson & Knopoff model (Hutchinson & Knopoff 1978) for two standardized synthetic complex-tones distant by that interval from each other, with the same loudness and the same harmonic spectrum containing the first 10 harmonics with every higher harmonic decreasing 3 dB in loudness from the previous one, with the roughness reading being taken from the transposition of the interval which has its lowest note placed at C4 (middle C, with its fundamental tone at 261.626 Hz), an optimal pitch height location of our hearing range. These standardization parameters were chosen so that the roughness readings would comparatively agree with commonplace historical notions of Music Theory.

The Tn-roughness is an abstraction of the roughness potential of a harmonic Tn-type, derived from the sum of its constituent roughness-classes according to the actual configuration in which the intervals appear in the normal form of the entity, which represents its most packed voicing, therefore the point of maximum roughness the Tn-type can acquire. The result can be normalized and rounded so that the maximum Tn-roughness possible is 15.0 (for the entire aggregate) and the minimum is 0.0 (the Tn-type (0)), with the major and minor triads yielding the value of 1.0 (Tn/TnI-type [0 3 7]).

Although a Tn-type is not usually meant to be played simultaneously, its Tn-roughness serves to measure the maximum amount of potential sensory consonance/dissonance stored inside it. With this in mind, the Tn-roughness can be used to reveal curious facts about well-known sets. For example, some of the most traditional musical structures historically in use for centuries have the least amount of stored sensory dissonance: the set type of cardinality 3 with the smallest Tn-roughness is the pair (0 3 7) & (0 4 7), the minor and major triads; the Tn-type of cardinality 5 with the smallest Tn-roughness is (0 2 4 7 9), the ubiquitous pentatonic scale; the Tn-type of cardinality 6 with the smallest Tn-roughness is (0 2 4 5 7 9), which is the medieval hexachord (ut-re-mi-fa-sol-la); and the Tn-type of cardinality 7 with the smallest Tn-roughness is (0 1 3 5 6 8 10), the common diatonic major/natural-minor scale. These findings seem to concur with observations previously made by David Huron using different methods (Huron 1994).

Tonicity and roots, phonicity and vertexes, and azimuth.

The concepts of tonicity and phonicity were first worked in detail by the theorist Arthur von Oettingen (Oettingen 1866). These concepts played an important part in the construction of the Harmonic Dualism of 19th century Music Theory (see Mickelsen 1977), and have served to define the borders between the ideas of major and minor, applying a kind of gender marking to pitch-class sets. Oettingen's concepts of tonic root and phonic overtone, which established a dualist origin for the perfect triads in a reasonable way without resort to an undertone series, can be recast today in a very interesting manner to define and separate the Tn-types into three genders: major, minor, and neutral.

Tonicity is the measurement of the extent to which a sonority seems to spring from a common fundamental tone: the root, or tonic root; it is the extent to which the sonority contains pitch-classes whose overtones are bound together by a common fundamental tone. In a way, it is the same idea psychoacousticians call tonalness: "the degree to which a sound has the sensory properties of a single complex tone such as a speech vowel" (Terhardt apud Parncutt 1989: 25). The root of a harmonic combination can then be defined as the note whose fundamental tone is related by unison or octave to the best candidate for a common fundamental tone for the entire sonority. The tonicity can be objectively quantified from Richard Parncutt's model of the perceptual root of a chord (Parncutt 1988 and 1997), which generates a table of root-saliences. In the model of tonicity I am proposing, the following assumptions are considered: a) the tonicity is directly proportional to the fraction of the total root-salience points generated by the set that was concentrated on the root (criterion of strength) ; b) the tonicity is directly proportional to the amount a root surpasses the second biggest root-salience value (criterion of segregation); and c) the tonicity is inversely proportional to the number of roots in the set (criteria of uniqueness). The calculated value can be normalized at the end so that the Tn-type (0) yields a tonicity value of 100 (100% tonic).

Phonicity is the measurement of the extent to which a sonority produces a common overtone: Oettingen's phonic overtone (Oettingen 1866), which I am proposing to have renamed as vertex, or phonic vertex; it is the extent to which the sonority contains pitch-classes that relate to different fundamental tones bound by a common overtone. The vertex of a harmonic combination can then be defined as the note whose fundamental tone is related by unison or octave to the best candidate for a common overtone for the notes of the sonority. In my model, the phonicity can be objectively quantified in the same manner as tonicity, but from an inverted version of Parncutt's table of root-saliences, which then becomes a table of vertex-saliences. The same assumptions made in regards to the calculation of tonicity are applied here as well.

In a similar vein as in Oettingen's works, sets which yield more tonicity than phonicity will belong to the major gender, and vice-versa for the minor gender. A set of the neutral gender will have equal values for tonicity and phonicity. I am proposing the name Azimuth as the measurement of a set's bias towards a tonic or a phonic nature. As a convention, this bias can be represented in angle degrees by an eastbound azimuth value with a positive sign for a sonority with tonic nature, and by a westbound azimuth value with a negative sign for a sonority with phonic nature. In this way, we can represent the biggest tonic azimuth possible with the value +90º (completely eastbound, which occurs for Tn-type (0 4 7), the major triad), and the biggest phonic azimuth possible with the value -90º (completely westbound, which occurs for Tn-type (0 3 7), the minor triad). Neutral sonorities will have a null azimuth (0º). Since the quantification of tonicity and phonicity are based on similar measurement principles, but implemented in a symmetrically opposed manner, the Azimuth can be calculated by simply subtracting the phonicity from the tonicity and normalizing the result so it fits into the desired range in degrees (-90º to +90º).

Due to the inversional relationship existent between tonicity and phonicity, the tonicity value of a Tn-type will equal the phonicity value of its inversion, and vice-versa. If a Tn-type has a positive value azimuth, its inversion will have an equal bias towards a phonic nature, bearing the same absolute azimuth value of its pair, but with the opposite sign.

Vicinity and Commonality.

Vicinity can be defined as the measurement of the amount of common tones between two sets (see "*parenté vicinale*", Costère 1954). Vicinity relationships play an important role on the assessment of possibilities of contrapuntal development of harmonic progressions, revealing the obvious connections between the notes of different harmonic combinations. The construction of invariance matrices, a technique taken from Set Theory, is highly useful for the fast mapping of vicinity relationships.

Commonality is the assessment of the similarity between the harmonic spectral contents of two pitch-class sets; it is a measure of how similar these two sets sound. Of course, vicinity plays an important part in commonality, but unlike the former, the later considers overtone coincidences weighted according to their perceptual hierarchies, not just note coincidences. To calculate it, we start with an ideal average measurement of the spectral contents of a pitch-class set, represented here simply by its phonic table of vertex-saliences, which I defined, as seen earlier on, as an inverted version of Parncutt's table of root-saliences. The commonality factor for two pitch-class sets can then be assessed by the summation of the amount of vertex-salience points in common between the two sets for each chroma individually, weighted according to the vertex-salience importance that chroma has in each of the two sets. The calculated value can be normalized by dividing the above calculation by the maximum commonality value possible, which is the commonality between the pitch-class set with the biggest number of items and itself, further converted to a percentage-like scale multiplying it by 100. Thus, we have between two identical pitch-class sets a commonality value of 100%. Commonality is of key importance for the assessment of sets with close-by sonorities which could be potentially useful for substituting for each other in contexts of harmonic prolongation (for the issue of prolongation in non-tonal contexts, see Straus 1987). It can also be used for assessing the size of the perceptual "bump" that would occur on the passing between two different harmonies. Commonality is also important for the concept of gravitational flow, as it defines one of the "shortest distance" relationships (Webern 1933 and Costère 1954), as it is going to be seen later on.

Gravitational flow.

The matter of the attractive effect some pitches exert to others has often been posited as an important piece in the puzzle of generating logical and coherent harmonic motion from one sonority to another. This is a force sprung from properties of the sound matter itself, independently of the composer's will:

> "Having reached this point beyond classical tonality, it is no less indispensable to obey, not new idols, but the eternal necessity of affirming the axis of our music and to recognize the existence of certain poles of attraction. Diatonic tonality is only one means of orienting music toward these poles. The function of tonality is completely subordinated to the force of attraction of the pole of sonority. All music is nothing more than a succession of impulses that converge toward a definite point of repose." (Stravinsky 1956: 35).

> "One can also take the view that even with us [Schoenberg and his pupils] there is still a tonic present – I certainly think so (...)." (Webern 1933: 39).

Edmond Costère tried to map these polarization pathways by means of his "Law of The Cardinal Gravitation of Sounds" (Costère 1954), which posits that a pitch-class tends to either remain static or to flow towards another pitch-class using the "shortest distance" method. This "shortest-distance" method can be defined in terms of pitch proximity (the leading-tone effect, represented in 12-tone equal temperament by the minor second) or in terms of pitch commonality (the diapente effect, represented in 12-tone equal temperament by the perfect fifth). Thus, a pitch-class of a set would have five cardinal notes, which would entertain cardinal attractions among themselves: the pitch-class itself, its superior and inferior leading-tones, and its superior and inferior diapentes.

Costère's theories, which give marvelous insights even into traditional tonal harmonic practice, give rise to an intricate system of classifications and properties which he then uses to classify and compare all 351 Tn-types. The critical revision of Costère's theories should play a decisive role on this Harmony treatise project of mine, unabashedly influenced by his "*Lois et Styles (...)*" (Costère 1954).

Symmetry.

If the pitch-classes of a set form the same configuration of intervals both under and over a certain pitch point (which is called the symmetry axis), the set is considered to be symmetrical to itself (see Costére 1954). Symmetry reflects in a very interesting manner on tonicity and phonicity, for symmetrical sets always have a null azimuth and can be said to be genderless or neutral sets. Symmetrical sets also generate symmetrical gravitational flow tables, and their root-salience and vertex-salience tables are symmetrical to each other, all about the very same axes of symmetry. To each pitch-class in the set there is a "double", called by Costère its "relative", which corresponds to its inverted counterpart. For any symmetrical set, the pair of relatives share the same value for gravitational flow and root/vertex-saliences, but with reversed polarity (major turning minor, and vice-versa). If an axis of symmetry is coincident with one of the pitch-classes of the set, this pitch-class constitutes a center of symmetrical intervallic articulation, the "median note" (Costère 1954).

The symmetries of a pitch-class set also generate relationships of total invariance with at least one of its inversions and sometimes also with some of its transpositions. This is the case with the famous modes of limited transposition (Messiaen 1956). From the 351 Tn-types possible, 95 are symmetrical (see Costère 1954). Examples of how symmetries and axes of symmetry can play a major structural compositional role can be found on several compositions by Béla Bartók (see Cohn 1988).

The structure of the treatise and its thesaurus.

The treatise should include a thesaurus, which will show, in an easily searchable manner and in comparative terms, all 351 Tn-types with information and descriptions such as its normal form, interval vector, inclusion relationships (complementary set, subset and superset relationships), as well as its transpositions, inversions, relative and isomeric relationships, localizing and measuring its roots, vertices, gravitational-flow poles, Tn-roughness, tonicity, phonicity, azimuth, symmetry relationships, and showing in a concise way the commonality and vicinity (invariance) relationships entertained between its various transpositions and inversions, alongside with its gravitational flow mappings. There should also be a diagnostic list with the interpretation of all data shown, according to a system of classifications of properties and features based mainly on Costère's research and on Set Theory concepts. The result will be a text consisting of: a) theoretical chapters, which would define, explain, and discuss all the theoretical aspects concerning the properties here mentioned, giving precise mathematical models for their quantization and illustrated by examples taken from music literature; and b) a thesaurus with one full-page entry for every Tn-type containing all information on it, with tables, charts, listings, and diagnostics.

The formulation of the mathematical models for all the properties here presented are currently being formalized in detail in several different articles pending publishing and tested for their accuracy and relevance. I have also been developing a computer software which implements the entire theory to generate the pages of the thesaurus with a clear and high-quality design. Given the amount of information and calculation involved in the thesaurus, my idea is to avoid human errors to the maximum extent possible regarding calculations and typos. An example of the thesaurus page for Tn-type (0 1 3 7) as generated by the current prototype of my software can be seen in figure 1.

The usefulness of such a text and thesaurus for composition and for analysis is also being currently tested in my own compositional work, in the courses I teach on Composition and Analysis, and in discussion with peers, which constantly give me new feedback, positing new questions and setting new directives and directions. With every new alteration to the theoretical models and software, a new pdf version of the entire thesaurus can be generated for the continuation of the testing and research activities. Whenever these sketches acquire the status of a cohesive and useful theory, the compilation of the treatise for proper publication should start to be viable. It is my hope that the utility of this project has already been somewhat adumbrated by the theoretical discussions here presented.

THESAURUS - 115 -

(0 1 3 7)

interval vector: <1 1 1 1 1 1>
Tn/TnI type: [0 1 3 7]
complementary: (0 2 3 4 6 7 8 9)
isomers: (0 1 4 6) (0 2 5 6)

Tn–roughness: 4.06 fusion: 1.18
tonicity: 9.38 phonicity: 33.75 •

Costère number: 21 13 14 i = 0
Forte number: 4–Z29

azimuth: –87.21°

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root:	***15***	11	(2)	***15***	(0	8	5)	10	(8	5	1	4)
vertex:	10	12	(6)	11	(3	6	0)	***18***	(5	1	7	5)
cardinal:	***3***	2	(***3***)	1	(1	1	2)	2	(***3***	0	1	1)
tonal M:	(6	6	5	4	5	4	5	6	***7***	3	5	4)
tonal m:	6	(6	4	4	4	***7***	4	6	5	4	4	6)
transpositional:	8	(***9***	5	5	5	***9***	6	***9***	5	5	5	***9***)
inversional:	(5	8	8	***10***	4	6	5	8	8	8	4	6)
Tn invariance:	***4***	1	1	1	1	1	2	1	1	1	1	1
TnI invariance:	1	***2***	***2***	***2***	***2***	0	1	***2***	***2***	0	***2***	0

4
items

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Asymmetrical
relative: (0 4 6 7)

integrally tonal–imitative
0m [ab=bA]

Cardinally Transitive
with balanced cardinal pole
cardinal poles: 0 (2) (8)

Tonal Minor
Tonally Explosive
Tonic m
tonal poles: (8M) (5m)
intrinsic tonic poles: 0m

COMMONALITY

	0	1	2	3	4	5	6	7	8	9	t	e
Tn:	100	28	36	32	36	39	***54***	39	36	32	36	28
TnI:	35	47	43	51	44	24	38	57	54	20	***63***	23
(0 4 7):	***53***	31	13	51	15	30	31	26	46	24	20	29
(0 3 7):	***79***	23	15	28	25	34	28	31	30	22	28	12

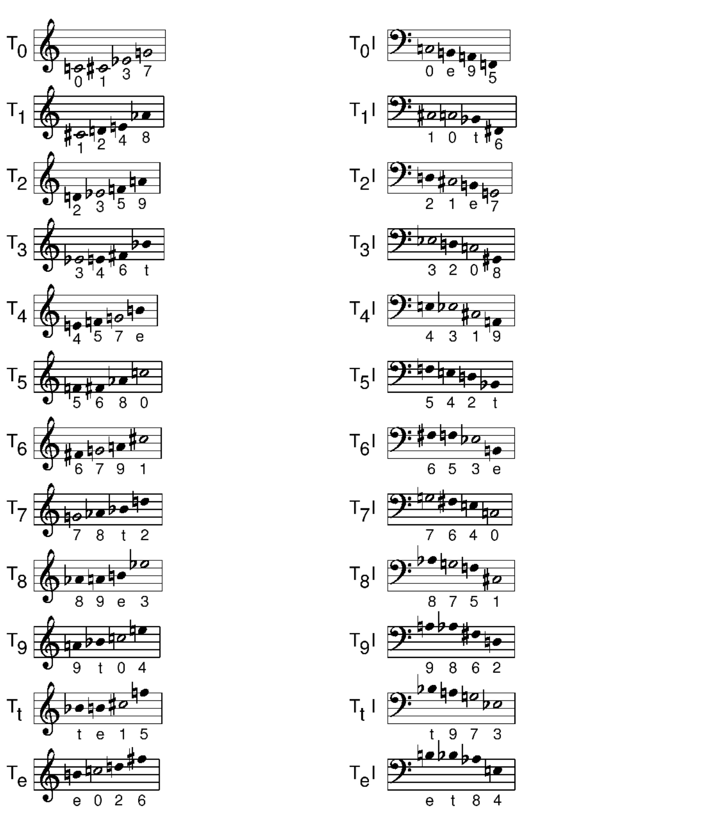

Fig. 1: The thesaurus page for Tn-type (0 1 3 7), as generated by the current prototype of the modeling software.

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